



A Statistical Regression Analysis of Financial Time Series Using ARIMA Models

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ABSTRACT

To take a sound strategic investment decision, prediction of financial markets becomes indispensable. The time series forecasting finds application in various traditional domains like the Auto-Regressive Integrated Moving Average (ARIMA) model as the model fully leverages its underlying capability of capturing trends and patterns in the historical data. This paper discusses the ARIMA model for forecasting life via torrent in the financial market and employs stock market index values from the Bombay Stock Exchange (BSE). The study revealed that ARIMA works particularly well for short-term forecasting, which gives essential market trend indications. This proves its usefulness for financial forecasting and also offers some valuable observations to investors and analysts wishing to study market behavior and improve forecasting accuracy.

Keywords: Financial Market Prediction, ARIMA Model, Time Series Forecasting, Short-term Prediction, Market Movements, Stock Index Data.

I.INTRODUCTION

According to Karamouz and Araghinejad (2012), ARIMA models are essentially dynamic and therefore completely unsuitable for reconstructing missing values. In another offshoot, Balaguer et al. (2008) studied the joint application of time series models, in particular ARMA and ARIMA, and artificial neural networks in different hydrological domains. In thiswise, Toth et al. (2000) utilized a combination of artificial neural networks and ARMA models to forecast rainfall occurrences. In much the same way, Mohammadi et al. (2005) applied artificial neural networks, regression techniques, and ARMA models to predict inflow to the Karaj reservoir by snowmelt. Chegini (2012) noticed the highest inflows occurred during the spring as a result of snow melting after winter thawing.

A significant contribution to the modeling and forecasting of financial markets using ARIMA techniques has been from M. Khasel et al. (2009), C. Lee & C. Ho (2011), and M. Khashei et al. (2012). Itimi et al. (2018), on the other hand, studied the positive risk premium theory in stock indices where it was revealed that higher returns are generally expected from investments with greater risk. Their outcomes inferred that even though highly volatile and relatively underdeveloped, the Nigerian stock market still reacts to macroeconomic shoves.

ONOUHA (2018) carried out deeper studies on predictability of stock market and set his sights on GCC (Gulf Cooperation Council) countries with a view to analyzing crude oil price-stock returns nexus. The study revealed that oil-based model forecasts outperformed those relying on traditional time series techniques, namely, AR, MA, ARMA, and ARIMA, based on the methodology of Wasteland et al. (2012,



2015). This conclusion was very much valid for both in-sample and out-of-sample forecasts and uniformly across different oil price indices (Brent and WTI) and diverse forecast horizons (30 and 60 days). And according to Mohammed Nafie and Alkesh (2019), the growth of the economy of Oman is expected at a steady rate of 3% in real terms. The forecast states that Oman is going to grow the most out of all GCC nations in 2019 which translates to a good business environment for investments, both domestic and foreign.

II.RELATED WORK

Augmented Dickey-Fuller (ADF)

The Augmented Dickey-Fuller (ADF) test is a widely used statistical method for testing whether a time series is stationary. Stationarity is really, really important in the world of time series analysis because it implies no trends or seasonality affecting the data. A stationary time series has a stable mean and variance over time, making it more appropriate for predictive modeling. Seasonal patterns and trends can really knock things off track at various points in time, so ensuring the data is stationary is half the battle in developing effective models that will forecast future values accurately.

The methods of testing for stationarity are numerous and include the W-D Test, Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF), Ljung-Box Test, t-Statistic Test, and KPSS-Test. The above tests collectively help determine whether or not a time series is stationary. ADF test is by far a foremost unit root test that's especially important to confirm whether or not stationarity prevails in a dataset. It seeks a static relationship, establishing a firm base for subsequent modeling and predictive examination.

These tests are crucial tools in time series analysis, allowing researchers to systematically evaluate and verify the stationarity of data. Applying these robust statistical methods improves the reliability of future analysis, in that models and forecasts are built on a firm understanding of the time series behavior. Employing these methods demonstrates the meticulous and diligent nature required for time-dependent data analysis, particularly in areas where proper forecasting and credible models are critical.

$$\Delta\lambda_t = \alpha_0 + \alpha_{2t} + \sum_{i=0}^k \beta \Delta\lambda_{t-1} + \epsilon_t$$

λ_t : Represents the monthly index value of the individual stock at time t .

β : Denotes the coefficient to be estimated in the regression model.

k : Indicates the number of lagged terms included in the analysis.

t : Refers to the trend term, capturing any linear trend in the time series.

α_2 : Represents the estimated coefficient associated with the trend term.

α_0 : Stands for the constant term in the model.

ϵ_t : Refers to white noise, representing random error terms assumed to be independently and identically distributed with a mean of zero and constant variance.



III.METHODOLOGY

Box-Jenkins Methodology

The Box-Jenkins methodology is a systematic approach to time series forecasting, primarily using the ARIMA (p, d, q) model. This model, developed by George Box and Gwilym Jenkins in 1970, builds upon the Autoregressive Moving Average (ARMA) model, incorporating differencing techniques to handle non-stationary data. The methodology follows a structured process to identify the most suitable ARIMA model for a given dataset, making it a widely used technique in time series analysis.

Components of the ARIMA Model

Autoregressive (AR) Process of Order p:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \epsilon_t$$

In this equation:

Y_t is the value of the time series at time t.

μ is a constant term.

$\phi_1, \phi_2, \dots, \phi_p$ are the autoregressive coefficients representing the influence of past observations.

ϵ_t is a random error term, also known as white noise.

Moving Average (MA) Process of Order q:

$$Y_t = \mu - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \dots - \theta_q \epsilon_{t-q} + \epsilon_t$$

Here, $\theta_1, \theta_2, \dots, \theta_q$ are the moving average coefficients, and past error terms influence the current value of the time series.

General ARIMA Model (p, d, q):

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \dots - \theta_q \epsilon_{t-q} + \epsilon_t$$

Y_t represents the differenced time series, meaning it has been adjusted to remove trends and achieve stationarity.

The d in ARIMA (p, d, q) refers to the number of times the series was differenced.

ϵ_t is a random error term, assumed to be independently and normally distributed with zero mean and constant variance.

The coefficients ϕ and θ are estimated during the model-fitting process.

Model Selection and Evaluation :

To determine the best-fitting ARIMA model, analysts use various statistical tests and criteria, including:

- Box-Ljung Q Statistic: Checks whether the residuals (errors) of the model are randomly distributed.
- R^2 (Coefficient of Determination): Measures how well the model explains variability in the data.



- Root Mean Square Error (RMSE): Quantifies the accuracy of predictions by calculating the average squared differences between actual and predicted values.
- Akaike Information Criterion (AIC) & Bayesian Information Criterion (BIC): Help identify the best model by balancing complexity and accuracy.

Performance of the Models

Mean Absolute Percentage Error (MAPE):

Mean Absolute Percentage Error (MAPE) is a commonly used statistical measure that evaluates the accuracy of a forecasting or predictive model. It represents the average percentage difference between the actual and predicted values, making it a widely accepted metric in business analytics, demand forecasting, and time series analysis. Since MAPE expresses error as a percentage, it allows easy comparison across different datasets and industries.

The formula for calculating MAPE is:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \times 100$$

Where:

Y_t = Actual observed value at time t

\hat{Y}_t = Predicted or forecasted value at time t

n = Total number of observations

Interpreting MAPE Values:

- 0% MAPE → Perfect prediction (no error).
- Less than 10% → Highly accurate forecast.
- 10% – 20% → Good accuracy.
- 20% – 50% → Moderate accuracy.
- More than 50% → Poor accuracy, indicating unreliable forecasts.

MAPE is a valuable tool for assessing the performance of predictive models, especially in industries that rely on accurate forecasts. However, for a more comprehensive evaluation, it is often used alongside other metrics such as Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

Mean Absolute Scaled Error (MASE):

The Mean Absolute Scaled Error (MASE) is a widely used metric for evaluating the accuracy of forecasting models. Unlike traditional error measures like Mean Absolute Percentage Error (MAPE), which can struggle with zero or near-zero values, MASE provides a standardized error measurement that allows for fair comparison across different datasets and time series. It is particularly beneficial when dealing with seasonal data or when comparing models across multiple forecasting problems.

The MASE formula is:

$$MASE = \frac{\sum_{t=1}^n |Y_t - \hat{Y}_t|}{\frac{1}{n-1} \sum_{t=2}^n |Y_t - \hat{Y}_{t-1}|}$$

Where:



Y_t represents the actual observed value at time t .

\hat{Y}_t is the predicted value at time t .

n is the total number of observations.

The denominator represents the mean absolute error of a naïve forecast, which assumes that each value is simply equal to the previous value.

Interpreting MASE Values:

- $MASE = 1 \rightarrow$ The forecasting model performs similarly to a naïve prediction.
- $MASE < 1 \rightarrow$ The model is more accurate than a simple naïve forecast.
- $MASE > 1 \rightarrow$ The model performs worse than a naïve forecast, meaning a basic method would have been more effective.

MASE is a powerful and reliable metric for assessing the performance of forecasting models. It provides a more balanced evaluation compared to traditional error measures, making it particularly useful for real-world applications. However, it is best used in combination with other accuracy metrics, such as Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), to get a comprehensive understanding of a model's effectiveness.

Akaike Information Criterion (AIC):

The Akaike Information Criterion (AIC) is a widely used statistical tool for model selection in time series analysis, regression, and machine learning. It helps identify the most suitable model by balancing the trade-off between accuracy and complexity. AIC discourages overfitting by penalizing models with excessive parameters, ensuring that the chosen model is both effective and efficient.

AIC Formula

$$AIC = 2k - 2\ln(L)$$

Where:

k = The number of estimated parameters in the model

L = The maximum likelihood function

$\ln(L)$ = The log-likelihood of the model

Interpreting AIC Values:

Smaller AIC values mean a better fit with lower complexity.

A higher AIC suggests that the model might be overfitting or not fitting the data adequately.

Although it compares well between models (as it is a relative measure of accuracy), AIC does not supply an absolute measure.

AIC is a useful tool that helps create the best model without overcomplicating it. Choosing the model with minimum AIC allows the analysts to maintain a trade-off between accuracy and simplicity. Nonetheless, utilizing AIC with other measures provide a more thorough characterization of the goodness of fit of the model such as using Metrics such as Bayesian Information Criterion (BIC) and Mean Squared Error (MSE).



IV.RESULTS

Null Hypothesis : Time series is non-stationary (has a unit root)

As an alternative, Our null hypothesis states that the time series is non-stationary (has a unit root).

To ensure the applicability of ARIMA modeling, we begin by testing for stationarity using the Augmented Dickey-Fuller (ADF) test. The null hypothesis asserts that the series possesses a unit root, indicating non-stationarity.

Interpreting the ADF Test Results

At Original Level

Test Statistic: -1.945

p-value: 0.6021

Since the p-value is greater than 0.05, we fail to reject the null hypothesis, suggesting that the time series is non-stationary in its original form.

After First Differencing

Test Statistic: -11.038

p-value: 0.0063

Here, the p-value is less than 0.05, indicating that we reject the null hypothesis. This confirms that the differenced time series is stationary, making it suitable for time series modeling.

Analysis of ARIMA Models Based on MAPE

Table 1: Results of Mean Absolute Percentage Error (MAPE) test

S.No	ARIMA Model	MAPE (%)
1	ARIMA(1,1,1)	4.82
2	ARIMA(2,1,1)	3.91
3	ARIMA(1,1,2)	4.35
4	ARIMA(2,1,2)	3.87
5	ARIMA(3,1,2)	4.26

Selecting the Best Model

Among all the models used for testing, has the smallest, so it is the best to choose.

The minor differences in MAPE for the various ARIMA models indicate that their forecast performances are all very similar.

As MAPE calculates error in terms of percentage, it is the best to choose the model with the lowest value.

Lowest MAPE: ARIMA(2,1,2) → Best short-term prediction performance.

Even by MAPE alone, appears to be the best model to use for forecasting because it has a lower error rate. Nonetheless, to verify robustness, other model selection criteria like Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Root Mean Square Error (RMSE) must also be taken into consideration before a decision is made.



Evaluating ARIMA Models Using MASE

The Mean Absolute Scaled Error (MASE) is an important measure for assessing the accuracy of prediction models. Contrary to common error metrics, MASE scales forecast errors relative to a naïve model, so it provides a sound metric for measuring predictive accuracy. A MASE measure less than 1 is an indicator that the model performs better than a simple forecast, whereas a measure greater than 1 represents less accuracy.

Table 2: Results of Mean Absolute Scaled Error (MASE) test

S.No	ARIMA Model	MASE
1	ARIMA(1,1,1)	0.87
2	ARIMA(2,1,1)	0.75
3	ARIMA(1,1,2)	0.82
4	ARIMA(2,1,2)	0.70
5	ARIMA(3,1,2)	0.79

Lowest MASE: ARIMA(2,1,2) → *More accurate than a naïve model.*

Selecting the Best ARIMA Model Using AIC

Akaike Information Criterion (AIC) is one of the most important metrics used for statistical model comparison. It indicates the best fit and penalizes high complexity to make the model efficient as well as accurate. The lower AIC value denotes a better model by balancing goodness of fit with simplicity.

Table 3: Results of Akaike Information Criterion (AIC) test

S.No	ARIMA Model	AIC
1	ARIMA(1,1,1)	295.82
2	ARIMA(2,1,1)	287.34
3	ARIMA(1,1,2)	290.11
4	ARIMA(2,1,2)	284.57
5	ARIMA(3,1,2)	289.26

Lowest AIC: ARIMA(2,1,2) → *Best model balancing fit and complexity.*

Conclusion on Stationarity:

The data is non-stationary at level but becomes stationary after first differencing. Therefore, the time series is integrated of order one, denoted as $I(1)$. This validates the use of an ARIMA($p,1,q$) model for forecasting.

Given this, the performance evaluation from MAPE, MASE, and AIC (see Tables 1–3) confirms that ARIMA(2,1,2) is the most robust model, showing the lowest forecasting error and optimal complexity, making it highly suitable for predicting short-term movements in the financial market.



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